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STRUCTURAL OPTIMIZATION OF DIFFERENT TRUSS MEMBERS USING FINITE ELEMENT ANALYSIS FOR MINIMUM WEIGHT

P. SATHEESH KUMAR REDDY & CH. NAGARAJU

Department of Mechanical Engineering, V.R. Siddhartha Engineering College,
Vijayawada, Andhra Pradesh, India

ABSTRACT

This paper aims to optimize and analyze a Warren, Pratt and Howe steel trusses and comparing Warren truss with other Pratt and Howe trusses. Existing structure steel trusses was initially optimized for minimum weight, in constrained with allowable stresses and deflections. The cross-sectional area of the truss members is taken as a design variable. Existing geometry and loading conditions of the truss are sized to mimic a real world environment. Every attempt was made to adhere to both state and federal regulations.

The structural steel trusses was optimized using the design optimization tool as first order optimization method in ANSYS and it is extended to compare for best suitable truss geometry for minimum weight. Mesh studies were performed on all ANSYS finite element models to ensure solution convergence. A comparison of the trusses was made by evaluating the minimum margin of safety in all truss members. To make a fair evaluation all trusses have identical geometries and loading conditions. The intent is to compare which truss is more efficient when constructing a truss.

Finally it is concluded that Warren truss is showing high stiffness to weight ratio over other trusses after optimization.

KEYWORDS: Warren Truss, Pratt Truss, Howe Truss, Steel, Optimization & Finite Element Method

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INTRODUCTION

Structural design is a branch of Engineering that deals with systems comprised from a set of structural members. These members may be characterized either truss or frame elements, connected by pinned or fixed joints. Common structures include truss bridges, frame buildings, race car and airplane space frames, crane arms, and power line truss towers. Structures may range in size from 1,450 ft. tall buildings weighing 222,500 tons down to bicycle frames weighing less than 10 lb.

Structural optimization has become a valuable tool for engineers and designers in recent years. Although it has been applied for over forty years, optimization in engineering has not been a commonly used design tool until high performance computing systems were made widely available. Structures are becoming lighter, stronger, and cheaper as industry adopts higher forms of optimization. This type of problem solving and product improvement is now a crucial part of the design process in today's engineering industry.

To better understand optimization of structures and the focus of this paper, two definitions must be stated. The first definition is that of the structure, including all implications and capabilities in the static analysis of such systems. The second definition applies to that of structural optimization, more specifically the optimization of size

and shape. These two primary definitions will hold true for the entirety of this research and are derived from McGuire (2000).

A structure is a set of nodes (vertices) that are connected by a set of elements (edges). This includes all plane (2D) and space (3D) truss and frame structures. Loads may be placed at nodes to exert a force or moment on the structure. Constraints may be placed at nodes to restrain the structure from translation or rotation caused by nodal loads. A valid structure must constrain at least all six degrees of freedom as a system, and over constraint will generally produce stiffer structures.

All elements are associated with a material defined by a minimum of two values: modulus of elasticity (E) and Poisson's ratio (μ). These values define the element's behaviour under static linear elastic loading conditions. Values used only for the optimization process include the element's yield strength (σ) and unit weight (Kg) or mass density (ρ). These values are used for stress limit comparison and structural mass, respectively.

Standard Form of the Optimal Design Problem

Design optimization seeks the best values of design variables, to achieve, within certain constraints, placed on the system behaviour, allowable stresses, geometry, or other factors, its goal ofoptimality defined by a vector of objective functions**,** for specified environmental conditions.

Mathematically, design optimization may be cast in the following standard form:

Statement of the Optimization Problem

An optimization or a mathematical programming problem can be stated as follows.

Find $\mathbf{X} = \{ x_1 , x_2 , x_3,.......... x_n \}$ which minimizes $f(\mathbf{X})$

subject to the constraints

$g_j(\mathbf{X}) \leq 0, j = 1, 2, . . . , m$

$l_j(\mathbf{X}) = 0, j = 1, 2, . . . , p$

where **X** is an n-dimensional vector called the *design vector*, $f(\mathbf{X})$ is termed the *objective function*, and $g_j(\mathbf{X})$ and $l_j(\mathbf{X})$ are known as *inequality* and *equality* constraints, respectively. The number of variables n and the number of constraints m and or pneed not be related in any way. The problem stated in Equation is called a *constrained optimization problem*. Some optimization problems do not involve any constraints andcan be stated as

Find $\mathbf{X} = \{ x_1 , x_2 , x_3,.......... x_n \}$ which minimizes $f(\mathbf{X})$

Such problems are called *unconstrained optimization problems*

FORMULATION OF OPTIMIZATION PROBLEM

- **Objective Function:** Minimize Weight

Member weight = Length x Member cross-sectional area x density

Truss weight = $\sum_{i=1}^{i=n} LixAix\rho$

Design Constraints: Stress & Nodal

Direct stress < 125 MPa

Nodal displacement < 25 mm

- **Design Variables :** Members cross-sectional Area

Starting Point : 50000 sq.mm

TRUSSES AND THEIR GEOMETRIES

A truss bridge is a bridge whose load-bearing superstructure is composed of a truss, a structure of connected elements forming triangular units. The connected elements (typically straight) may be stressed from tension, compression, or sometimes both in response to dynamic loads. Truss bridges are one of the oldest types of modern bridges. The basic types of truss bridges shown in this article have simple designs which could be easily analyzed by 19^{th} and early 20^{th}century engineers. A truss bridge is economical to construct because it uses materials efficiently. A truss is a simple structure whose members are subject to axial compression and tension only and but not bending moment. The most common truss types are Warren truss, Pratt truss and Howe truss. Warren truss contains a series of isosceles triangles or equilateral triangles. To increase the span length of the truss bridge, verticals are added for Warren Truss. Pratt truss is characterized by having its diagonal members (except the end diagonals) slanted down towards the middle of the bridge span. Under such structural arrangement, when subject to external loads tension is induced in diagonal members while the vertical members tackle compressive forces. Hence, thinner and lighter steel or iron can be used as materials for diagonal members so that a more efficient structure can be enhanced. The design of Howe truss is the opposite to that of Pratt truss in which the diagonal members are slanted in the direction opposite to that of Pratt truss (i.e. slanting away from the middle of bridge span) and as such compressive forces are generated in diagonal members. Hence, it is not economical to use steel members to handle compressive force.

Table 1: Existing and Allowable Data for Different Trusses

Design Parameters	Warren Truss	Howe Truss	Pratt Truss
Number of truss members	19	21	21
Max. Allowable Stress (MPa)	125	125	125
Max. Deflection (mm)	25	25	25
Total Weight of Truss members (kN)	14.45	16.71	16.75
No. of Joints	11	12	12
Cross sectional Area (Sq.mm)	50000	50000	50000

MATERIAL PROPERTIES

Most truss bridges are constructed out of structural steel but wooden truss bridges are not uncommon if the loading is minimal. In bridge designs that utilize these materials the stresses are computed and then compared with a material allowable. If the stress is too high a designer has only two choices. One is to increase cross sectional area and the other is to redesign the geometry of the truss to more evenly distribute load. Each of these choices has unfortunate tradeoffs. Increasing cross sectional area increases weight which adds additional loading the truss has to carry. It can also cause other geometric problems which may violate the design parameters of the bridge. Redesigning the truss geometry adds to the number of connections needed in the truss and possible points of failure. When these two options are not available the designer has no choice but to go for optimization which can lead to the need for larger or smaller cross

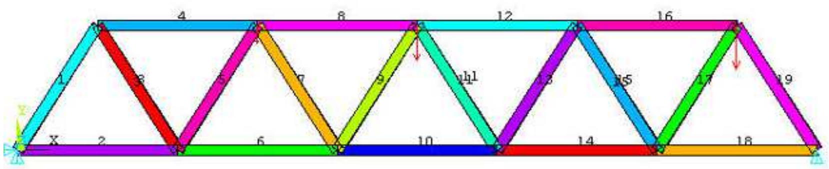
sectional area.

The truss element is independent of the second moment of area therefore only the normal stresses are of main concern. Circular cross sections are best for bearing normal stress however L-shape rolled sections are used in this study due to their wide use and ease of joining and are taken as per AISC manual with material as per ASTM-A36; Modulus of elasticity = 200 GPa, Poisson ratio = 0.29, Yield strength = 250MPa, Ultimate strength = 1930MPa and density = 7860 kg/m^3. As factor of safety taken for the design is 2, hence allowable stress is 125MPa. Analysis and optimization is performed using ANSYS software .

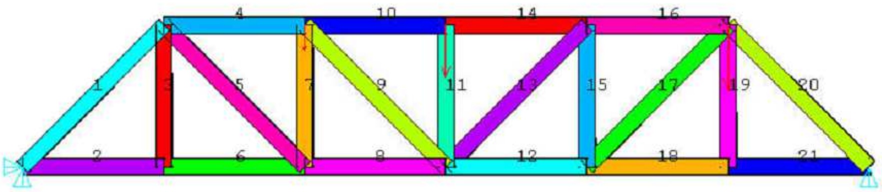
MODEL GEOMETRY, ELEMENT TYPE AND MESH

The CAD model of the truss was generated using nodes and elements in ANSYS. The members were modelled with nodes joining. Figure 1 shows this CAD model how they are connected to get a nineteen bar Warren truss, twenty one bar Pratt truss and twenty one bars Howe truss. The left end joints are pivoted and right end joints restrained in vertical direction. The vertical loads acting downward are with magnitudes 200kN, 400kN and 500kN at the similar joints for all trusses.

Warren Truss



Pratt Truss



Howe Truss

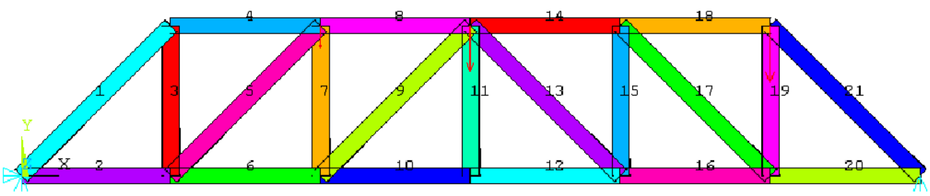


Figure 1: 2D ANSYS FEM Geometry

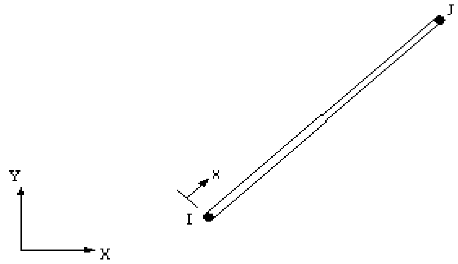


Figure 2: LINK-1 2D Spar

Once the CAD model was created an element type was selected. LINK 1 for truss members was chosen as the most appropriate elements for this analysis. Figure 2 shows a LINK 1 element.

LINK 1 is suitable for members with different area of cross sections as real constants

LOADS AND BOUNDARY CONDITIONS

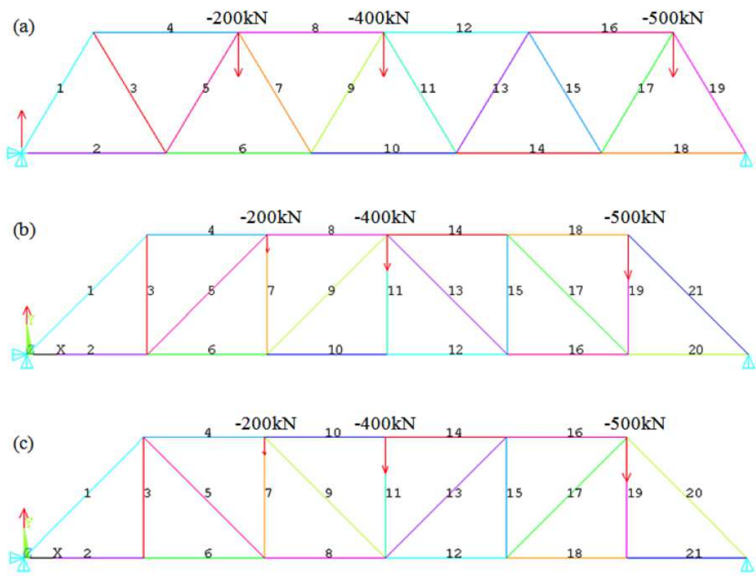


Figure 3: (a) Warren Truss (b) Howe Truss (c) Pratt Truss

The steel truss material was defined as linear isotropic truss material. The inputs for the isotropic material are elastic modulus and Poisson ratio. Loads were applied to the truss as per the free body diagram. The application point of these loads was at the nodes connecting the beams of the truss. The boundary conditions were set such that the truss is simply supported. This means that left node is fixed in space and right node is set as a pinned support. This translates to restraining left node from movement in the x, y, and z direction as well as a rotationally constraining it around the x and y axis. It also sets right node as restrained from movement in the y and z directions and the rotationally restrained are around the x and y axis.

RESULTS

The results of applying the analysis methodologies detailed in the previous sections are presented in the following section. Results of the different truss are presented along with optimization of each candidate member. The results of this study will help choose the best truss for the real time loadings.

The results of the 2D FEM are nodal displacements and stresses in each member in the axial direction. The distribution of stresses throughout this model did not vary significantly. Some of the members in compression and some of the members in tension. The only difference was the magnitude of the stresses in each truss. A colour plot of the 2D finite element model axial stress results for all truss members are shown in Figures.

Structural Analysis of Pre and Post Optimization of Truss Members

It is identified before optimization warren truss is with maximum nodal displacement 1.7855 mm where as in Pratt and Howe trusses are 2.264 and 2.224 mm respectively. After optimization maximum nodal displacement 18.99 mm where as in Pratt and Howe trusses are 20.39 and 20.31 mm respectively. These values are not violated the design constraint. This shows Warren truss is having high stiffness before optimization and after optimization. The difference in nodal displacement magnitude between the models can be seen in the summary of results in Table 2. It is identified that there are two zero force members in case of Pratt and one zero force member in case of Howe trusses which is not true in case of Warren truss as before and after optimization. After optimization stress constraint is no were exceeded with factor of safety 2 as shown in Table 3. It shows the real resistance of loads for Warren truss due to its geometrical structure is far better than Pratt and Howe trusses.

Table 2: Nodal Displacements among Different Trusses in mm

Node Joint	Pre – Optimization						Post - Optimization					
	Warren Truss		Pratt Truss		Howe Truss		Warren Truss		Pratt Truss		Howe Truss	
	Ux	Uy	Ux	Uy	Ux	Uy	Ux	Uy	Ux	Uy	Ux	Uy
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0.056	-0.993	0.083	-0.960	0.083	-1.042	1.420	-10.21	1.185	-8.403	1.206	-9.615
3	0.224	-1.653	0.166	-1.753	0.250	-1.878	2.876	-16.95	2.370	-14.43	2.406	-16.73
4	0.447	-1.655	0.333	-2.184	0.460	-2.224	4.339	-16.94	3.574	-19.20	3.616	-20.31
5	0.610	-1.066	0.506	-1.848	0.670	-1.923	5.793	-10.23	4.779	-14.46	4.827	-16.74
6	0.712	0	0.643	-1.202	0.843	-1.239	7.229	0	5.978	-8.450	6.028	-9.608
7	0.650	-0.513	0.780	0	0.980	0	6.508	-5.53	7.176	0	7.226	0
8	0.537	-1.406	0.724	-0.960	0.723	-0.959	5.067	-14.03	5.994	-8.403	5.999	-8.408
9	0.342	**-1.785**	0.557	-1.837	0.640	-1.834	3.607	**-18.99**	4.790	-15.62	4.792	-15.61
10	0.149	-1.409	0.347	**-2.264**	0.473	**-2.224**	2.148	-14.02	3.579	**-20.39**	3.592	**-20.31**
11	0.016	-0.642	0.137	-1.884	0.300	-1.886	0.701	-5.570	2.368	-15.60	2.391	-15.59
12			-0.035	-1.202	0.163	-1.203			1.162	-8.450	1.193	-8.460

Table 3: Axial Stresses among Different Truss Members

Element Number	Pre – Optimization			Post - Optimization		
	Warren Truss	Pratt Truss	Howe Truss	Warren Truss	Pratt Truss	Howe Truss
	Axial stress (MPa)	Axial stress (MPa)	Axial stress (MPa)	Axial stress (MPa)	Axial stress (MPa)	Axial stress (MPa)
1	-9.0963	-11.785	-11.785	-119.84	-120.48	-120.47
2	4.68	8.3333	8.3333	118.36	118.51	120.65
3	9.0963	**0**	8.3333	119.84	**0**	120.65
4	-9.36	-16.667	-8.3333	-120.05	-120.42	-120.65
5	-9.0963	11.785	-11.785	-119.84	120.48	-120.47
6	14.04	8.3333	16.667	121.33	118.51	120
7	4.4315	-8.3333	4.3333	117.73	-118.51	112.5
8	-16.32	16.667	-16.667	-121.66	120.42	-120
9	-4.4315	6.1283	-6.1283	-117.73	118.55	-119.8
10	**18.6**	**-21**	**21**	**121.9**	**-121.1**	**121.03**

Table 3 Contd.,

11	-4.898	-8	**0**	-118.12	-118.45	**0**
12	-16.08	17.333	21	-121.63	120.54	121.03
13	4.898	5.1854	-5.1854	118.12	120.15	-117.21
14	13.56	-21	-17.333	121.22	-121.1	-120.19
15	-4.898	-3.6667	3.6667	-118.12	-113.32	114.87
16	-11.04	-17.333	17.333	-120.61	-120.54	120.19
17	4.898	5.1854	-5.1854	118.12	119.97	-117.22
18	8.52	13.667	-13.667	119.67	119.83	-119.8
19	-16.56	**0**	3.6667	-121.54	**0**	114.87
20	-	-19.328	13.667	-121.84	119.8	-121.84
21	-	13.667	-19.328	119.83	-121.31	119.83

DISSCUSSION ON OPTIMIZED RESULTS OF TRUSSES

It is confirmed that optimized trusses are within the limits of design constraints, now it is require to analyze cross-sections of all the members may not be same, which is not necessary since all the members are not in a position to take heavy loads. Therefore, optimization tool in 2D ANSYS FEM was used to calculate optimum cross-sectional areas for each and every truss member in reducing weight of truss and their by reducing loads at joints causing reduction in stress levels. Area of cross-sections of all the members are taken as a design variables, allowable stress in the members are taken as a state variables along with weight of each members as objective function. It is identified that there is a big drop in weight of the truss and area of cross-sections making the truss optimized. The following graphs and figures show reduction in structural weight in proportion with cross-sectional areas of individual members.

Warren Truss

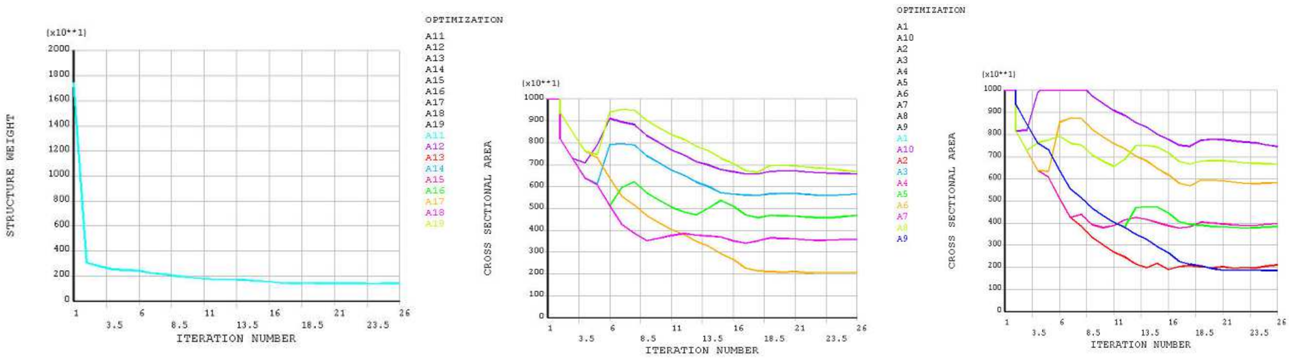


Figure 4: Iteration Wise Reduction in Structural Weight and Area of Cross-Sections for Warren Truss Members

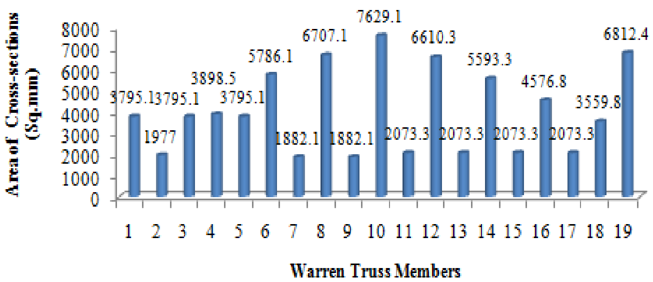


Figure 5: Optimized Cross-Sectional Area in Warren Truss Members with Existing 50000 Sq.Mm

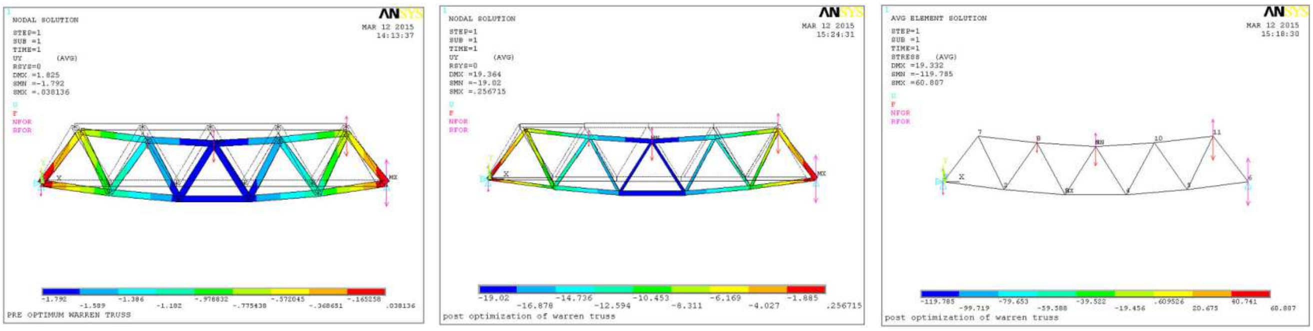


Figure 6: Pre & Post Optimum Finite Element Model and Results Obtained for Warren Truss

Pratt Truss

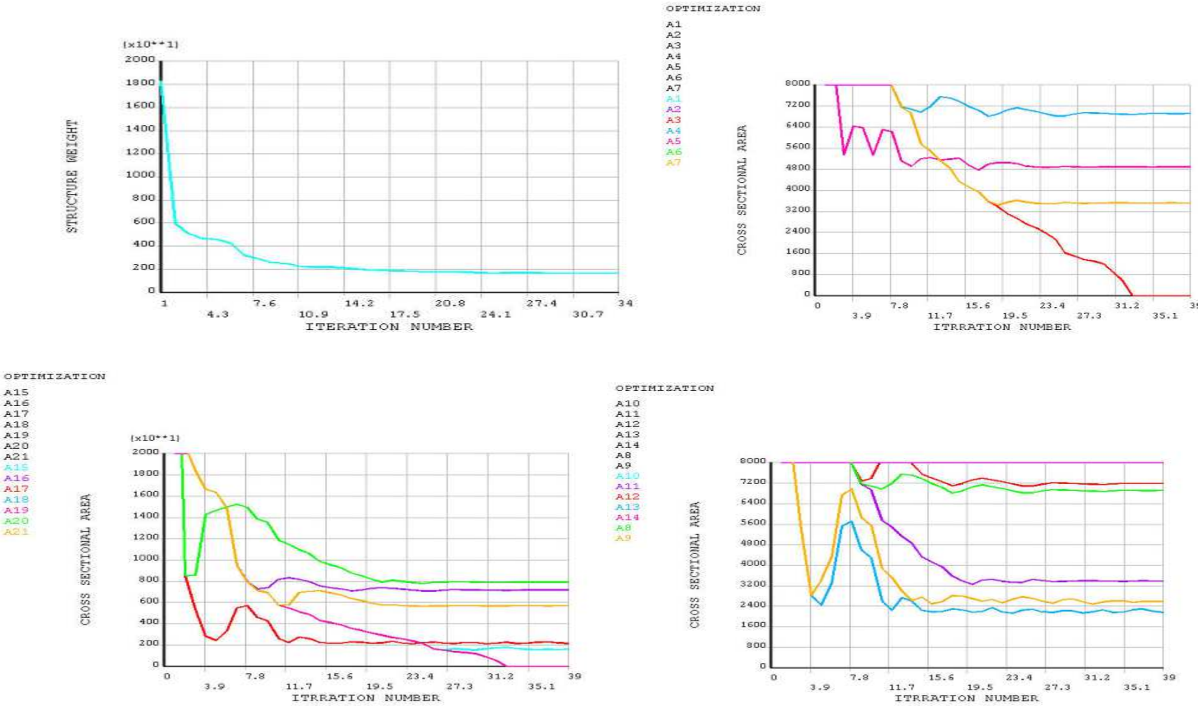


Figure 7: Iteration Wise Reduction in Structural Weight and Area of Cross-Sections for Pratt Truss Members

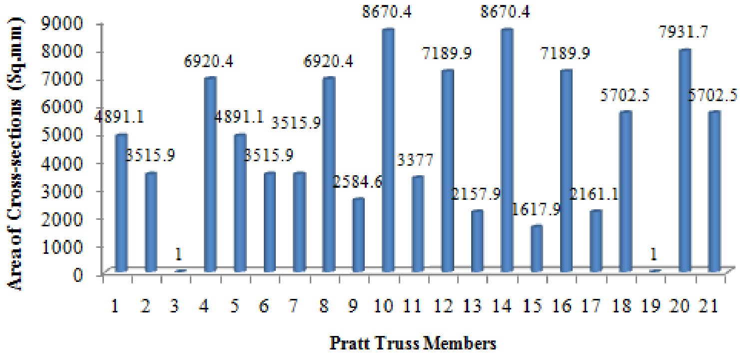


Figure 8: Optimized Cross-Sectional Area in Pratt Truss Members With Existing 50000 Sq.Mm

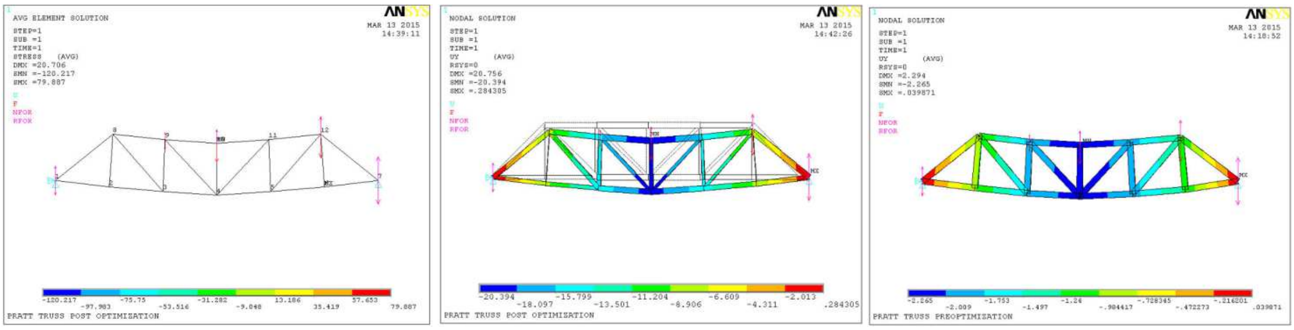


Figure 9: Pre & Post Optimum Finite Element Model and Results Obtained for Pratt Truss

Howe Truss

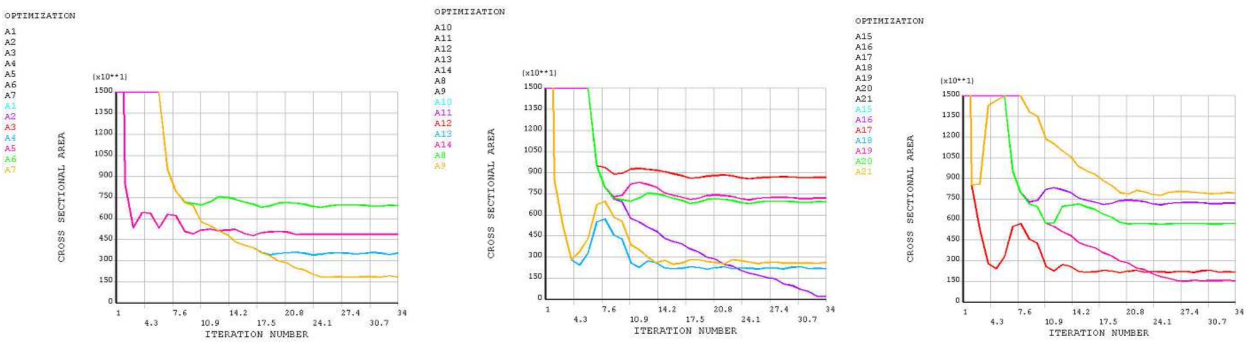


Figure 10: Iteration Wise Reduction in Structural Weight and Area of Cross-Sections for Pratt Truss Members

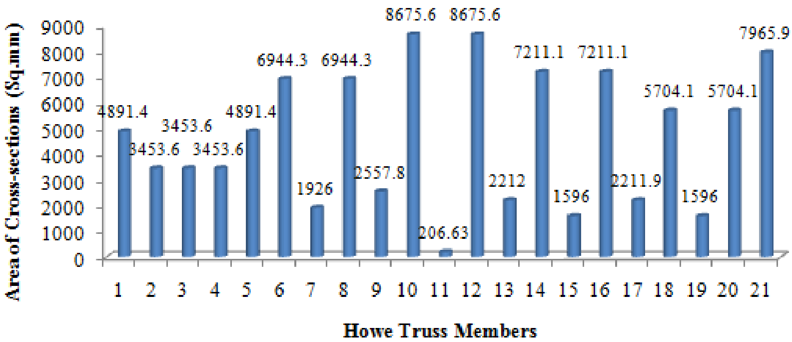


Figure 11: Optimized Cross-Sectional Area in Howe Truss Members with Existing 50000 Sq.Mm

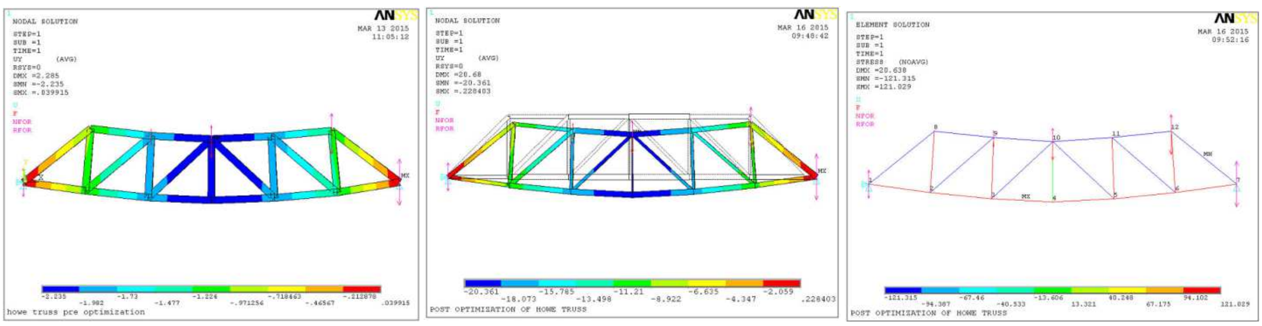


Figure 12: Pre & Post Optimum Finite Element Model and Results Obtained for Howe Truss

MARGIN OF SAFETY (M.S)

The ultimate goal of using optimization for the construction of a truss bridge is achieving a lower weight structure that has higher strength. Strength can be quantified by calculating a margin of safety in each truss member. Margin of safety is defined as in equation.

$$M.S. = \frac{Allowable\ Strength}{Actual\ Strength}$$

When the M.S. reaches a value less than or equal to one the structure will fail. An alternate definition is that it is equal to the factor of safety. This factor is used to compare different trusses and members among them. This is done by computing the M.S in every member of every truss model for axial direction. Then a comparison between the lowest axial M.S. in each truss is made. The truss with the highest M.S. is deemed the strongest and the one with the lowest is ranked as the weakest. This results in a simple quantitative parameter to compare between each truss model.

Steel Truss Margin of Safety

The intermediate alloy ASTM-A36 steel is an isotropic material. This means it has the same strength in every direction. The material allowable for this strength only varies with the type of stress applied. Table 4 details the material allowable for steel material in compression and tension.

Table 4: Allowable Stresses in Steel Material

ASTM-A36	Allowable (MPa)
$\sigma_{ty\ (Tensile)}$	250
$\sigma_{cy\ (Compressive)}$	250

The maximum axial stress in each member as computed was compared to the appropriate tension or compression material allowable. The member stresses used in this calculation can found in the previous section. The resulting axial M.S. is shown in Table 5. It is identified that margin of safety is greater than one for all the members after optimization, and is also greater than two for all the members with high factor of safety.

Table 5: Margin of Safety Calculations

Element Member	Warren truss		Pratt Truss		Howe Truss	
	Stress	M.S	Stress	M.S	Stress	M.S
1	-119.84	2.09	-120.48	2.08	-120.47	2.08
2	118.36	2.11	118.51	2.11	120.65	2.07
3	119.84	2.09	0	---	120.65	2.07
4	-120.05	2.08	-120.42	2.08	-120.65	2.07
5	-119.84	2.09	120.48	2.08	-120.47	2.08
6	121.33	2.06	118.51	2.11	120	2.08
7	117.73	2.12	-118.51	2.11	112.5	2.22
8	-121.66	2.05	120.42	2.08	-120	2.08
9	-117.73	2.12	118.55	2.11	-119.8	2.09
10	121.9	**2.05**	-121.1	2.06	121.03	2.07
11	-118.12	2.12	-118.45	2.11	0	---
12	-121.63	2.06	120.54	2.07	121.03	2.07
13	118.12	2.12	120.15	2.08	-117.21	2.13
14	121.22	2.06	-121.1	2.06	-120.19	2.08
15	-118.12	2.12	-113.32	2.21	114.87	2.18
16	-120.61	2.07	-120.54	2.07	120.19	2.08

Table 5 Contd.,

17	118.12	2.12	119.97	2.08	-117.22	2.13
18	119.67	2.09	119.83	2.09	-119.8	2.09
19	-121.54	2.06	0	---	114.87	2.18
20			-121.84	**2.05**	119.8	2.09
21			119.83	2.09	-121.31	**2.06**

WEIGHT REDUCTION IN TRUSSES

It is confirmed that optimized trusses are within high margin of safety or factor of safety, now it is require to evaluate the weight of the structural members as well as trusses. The below Table 6 shows weight reduction during iterations using optimization technique. Due to reduction in weight a lot of material can be saved which saves a lot of cost during construction .The first row shows weight reduction of warren truss using 26 iterations and the weight is reduced from 17471 kg to 1418kg.The second row shows weight reduction of Pratt truss using 39 iterations and the weight is reduced from182367 kg to 1670 kg. The third row shows weight reduction of Howe truss using 34 iterations and the weight is reduced from 18267 kg to 1676kg

Table 6: Weight Reduction and Convergence During Iterations

Iteration	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Warren	17471	3068	2776	2557	2501	2426	2242	2163	1996	1894	1801	1751	1737	1697	1636	1552	1455
Pratt	18267	5948	5078	4667	4605	4283	3217	2905	2587	2521	2255	2209	2207	2149	2045	1974	1925
Howe	18267	5948	5078	4667	4605	4283	3217	2905	2587	2521	2255	2209	2207	2149	2045	1974	1925

Iteration	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
Warren	1427	1450	1439	1437	1421	1416	**1414**	1416	1418	Converged...							
Pratt	1867	1842	1823	1814	1803	1772	1751	1736	1719	1713	1712	1716	1712	1697	1685	1673	**1670**
Howe	1867	1839	1806	1801	1776	1759	1714	1694	1701	1705	1703	1692	1695	1687	1677	**1676**	1677

Iteration	35	36	37	38	39	40	41
Warren	Converged...						
Pratt	1670.4	1673.1	1675.5	1670.5	1669.6	Converged...	
Howe	Converged...						

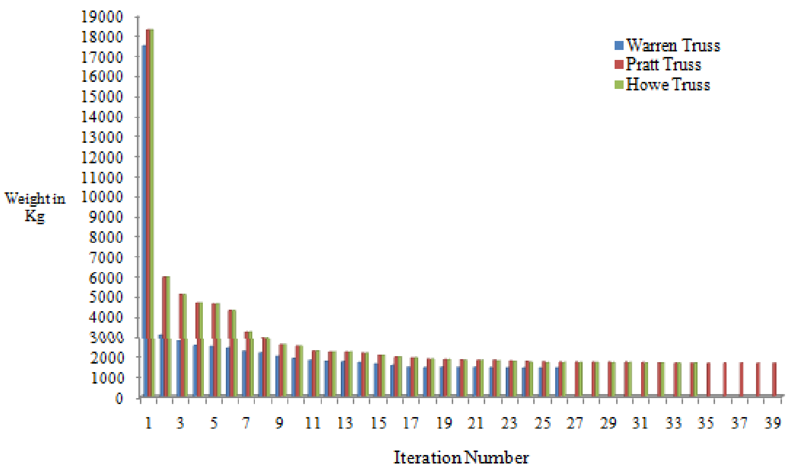


Figure 13: Weight Reduction With Iteration Number of Three Trusses

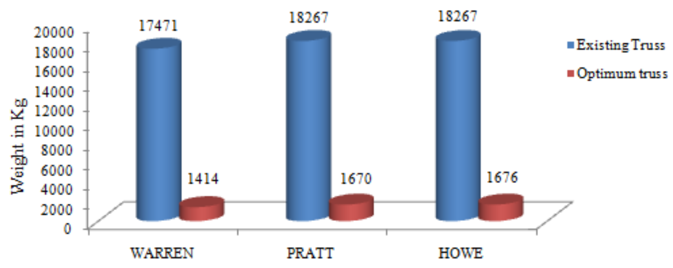


Figure 14: Pre And Post Optimum Weight of The Truss

Table 7: Saving in Weight Without Violating Design Constraint

Weight Saving in Kg	Existing Truss	Optimized Truss	Saving in Weight
Warren Truss	17,471	1,414	13,327
Pratt Truss	18,267	1,670	15,597
Howe Truss	18,267	1,676	16,691

CONCLUSIONS

- After the optimization Warren truss is with less weight compared to Pratt & Howe trusses for similar loading.
- It is identified that margin of safety is more or less equal in all trusses and also satisfied the design constraint.
- It is identified that nodal displacements at critical nodes are also more are less equal and in limits.
- It is identified that there are two zero force members in case of Pratt and one zero force member in case of Howe trusses which is not true in case of Warren truss. It shows the real resistance of loads for Warren truss due to its geometrical structure is far better than Pratt and Howe trusses.
- Construction of optimized 19-bar Warren truss is recommended when compared to Pratt & Howe trusses.

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